Hidden Conformal Symmetry of the Kerr-Newman Black Hole

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ABSTRACT: We investigate the hidden conformal symmetry of the 4-dimensional non-extremal Kerr-Newman (KN) black hole with the idea of the near-region Kerr/CFT correspondence proposed by Castro, Maloney and Strominger in arXiv:1004.0996[hep-th]. The near-region KN black hole is dual to a 2D CFT with left and right temperatures $T_L = (2M^2 - Q^2)/(4\pi J)$ and $T_R = \sqrt{M^4 - J^2 - M^2 Q^2}/(2\pi J)$. Furthermore, we reproduce the microscopic entropy of the KN black hole via the Cardy formula, which is in agreement with the macroscopic Bekenstein-Hawking entropy and precisely match the absorption cross section of a near-region scalar field in the KN black hole with the finite-temperature absorption cross section for a 2D CFT.

KEYWORDS: AdS-CFT Correspondence, Black hole.

Contents

1. Introduction

The study of rotating extremal black holes has been an interesting subject since the original work on a new conjecture called the Kerr/CFT correspondence [1], which states that near-horizon states of a four-dimensional extremal Kerr black hole could be identified with a two-dimensional chiral CFT on the spatially infinite boundary, and the central charge is proportional to the angular momentum of the black hole. Thus, one can derive the microscopic entropy of the four-dimensional extremal Kerr black hole by the Cardy formula. It is a remarkable fact that this correspondence does not rely on supersymmetry or string theory. It was initiated in the context of the 4D Kerr in general relativity, then extended to the higher-dimensional rotating solutions in supergravity and string theories [2]. More works on the Kerr/CFT correspondence and its generalization are listed in [3].

Recently, Castro, Maloney and Strominger (CMS) [4] presented a remarkable result that hidden conformal symmetry of the 4D non-extremal Kerr Black hole can be obtained by the study on the low-frequency wave equation of a massless scalar field in the near region of the Kerr black hole. A CFT with certain central charges and temperatures is dual to the corresponding Kerr Black hole. This evidence can be verified by interpreting the absorption cross section of the scalar in the near region as finite-temperature absorption section for a 2D CFT. Subsequently, with the idea of hidden conformal structure of the non-extremal black hole, Krishnan [5] has extended this method to include five-dimensional black holes in string theory. Furthermore, the non-extremal uplifted 5D Reissner-Nordstrom black holes have been investigated by Chen and Sun [6].

In 4D general relativity theory, the Kerr black hole can be considered as the uncharged Kerr-Newman (KN) black hole. So, we can mention that does the hidden conformal symmetry of the KN black hole exit, and if so, how? It is the aim of this article to further investigate the hidden conformal symmetry of the four-dimensional KN black hole in general relativity theory following the approach of CMS in [4]. We will study the wave equation of the near-region scalar field and the corresponding $SL(2, R)$ Casimir structure, and look for the agreement for absorption cross sections between the CFT and gravity.

The paper is organized as follows. In Sec. 2, we review the KN black hole and derive the near-region wave equation of a massless scalar in the KN background. Furthermore, with the definition of conformal coordinates, we obtain the $SL(2, R)$ Casimir structure of wave equation and reproduce the corresponding microscopic entropy of the KN black hole via the Cardy formula. In Sec. 3, we show that the absorption cross section for the near-region scalar field is dual to the finite temperature absorption cross section in a 2D CFT. The last section is devoted to conclusion.

2. Wave equation in the Kerr-Newman background

In this section, we study the symmetry of the near-region scalar wave equation in the KN black hole background. The KN solution describes both the stationary axisymmetric asymptotically flat gravitational field outside a massive rotating body and a rotating black hole with mass, charge and angular momentum.

In terms of Boyer–Lindquist coordinates, the KN metric reads

$$ds^2 = -\left(\frac{\Delta - a^2 \sin^2 \theta}{\varrho}\right) dt^2 - \left(\frac{2a \sin^2 \theta (r^2 + a^2 - \Delta)}{\varrho}\right) dt d\phi$$
$$+ \left(\frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\varrho}\right) \sin^2 \theta d\phi^2 + \frac{\varrho}{\Delta} \, dr^2 + \Sigma \, d\theta^2, \tag{2.1}$$

where Δ is the KN horizon function,

$$\Delta = r^2 - 2Mr + a^2 + Q^2,$$
$$\varrho = r^2 + a^2 \cos^2 \theta, \tag{2.2}$$

and the gauge field A is

$$A = -\frac{Qr}{\varrho}(dt - \sin^2 \theta d\phi). \tag{2.3}$$

Here a is the angular momentum for unit mass as measured from the infinity, and M and Q are the mass and electric charge of the KN black hole, respectively. The non-extremal KN black hole has the event horizon r_+ and the Cauchy horizon r_- at

$$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}. \tag{2.4}$$

In what follows, we will see it is convenient to write the entropy S_{BH}, surface gravities $\kappa_\pm$, Hawking temperature T_H, angular velocity Ω_H as :

$$S_{BH} = \frac{\mathcal{A}_+}{4} = \pi(r_+^2 + a^2),$$
$$\kappa_\pm = \frac{2\pi(r_+ - r_-)}{\mathcal{A}_\pm},$$
$$T_H = \frac{\kappa_+}{2\pi} = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)},$$
$$\Omega_H = \frac{4\pi a}{\mathcal{A}_+} = \frac{a}{r_+^2 + a^2}, \tag{2.5}$$

here, we use the subscripts "±" to denote the outer and inner horizons of the KN black hole.

Now, we consider the Klein-Gordon equation for a massless uncharged scalar field in the background of the KN black hole:

$$\Box \Phi = 0. \tag{2.6}$$

The wave function is written in the eigenmodes of the asymptotic energy ω and angular momenta m as

$$\Phi(t, r, \theta, \phi) = e^{-iwt + im\phi} R(r) S^\ell(\theta). \tag{2.7}$$

Then, the wave equation (2.6) separates into the angular part

$$\left[\frac{1}{\sin\theta} \partial_\theta(\sin\theta\partial_\theta) - \frac{m^2}{\sin^2\theta} + a^2\omega^2\cos^2\theta \right] S^\ell(\theta) = -K_\ell S^\ell(\theta), \tag{2.8}$$

and the radial part

$$\left[\partial_r \Delta \partial_r + F + G \right] R(r) = K_\ell R(r), \tag{2.9}$$

with

$$F = \frac{2am(Q^2 - 2Mr)\omega + a^2(m^2 - 4M^2\omega^2)}{r(r - 2M) + a^2 + Q^2} \\ + \frac{(Q^4 - 4M^2Q^2 + 8M^3r - 4MQ^2r)\omega^2}{r(r - 2M) + a^2 + Q^2}, \tag{2.10}$$

and

$$G = (r^2 + 2Mr + 4M^2 - Q^2)\omega^2. \tag{2.11}$$

For $a\omega \ll 1$, the angular equation (2.8) can be written as the spherical harmonic function equation and the eigenvalues K_ℓ is determined by

$$K_\ell = \ell(\ell + 1). \tag{2.12}$$

Following the argument of CMS in [4], we also consider the same near-region, which is defined by

$$\omega M \ll 1 \quad \text{and} \quad r\omega \ll 1. \tag{2.13}$$

Then using the condition $M^2 \geq a^2 + Q^2$ and the above equation, we can obtain

$$\omega Q \ll 1, \quad Q/r \ll 1. \tag{2.14}$$

With the conditions of (2.13) and (2.14), the term G in Eq. (2.9) can be omitted. Thus the radial part of the wave equation can be reduced to the following near region equation:

$$(\partial_r \Delta \partial_r + F) R(r) = K_\ell R(r). \tag{2.15}$$

Next, we show that Eq. (2.15) can be reproduced by the use of the $SL(2,R)$ Casimir operator. First, we introduce the conformal coordinates $(w^\pm, y)$, which is analogue of the definitions in [4–6]:

$$
\begin{aligned}
w^+ &= \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_R \phi - 2\lambda_R t}, \\
w^- &= \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_L \phi - 2\lambda_L t}, \\
y &= \sqrt{\frac{r_+-r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - (\lambda_R+\lambda_L)t},
\end{aligned}
\tag{2.16}
$$

where

$$
T_R = \frac{\kappa_+}{2\pi\Omega_H}, \qquad T_L = \frac{\kappa_+(\kappa_- + \kappa_+)}{2\pi\Omega_H(\kappa_- - \kappa_+)},
$$
$$
\lambda_R = 0, \qquad\qquad \lambda_L = \frac{\kappa_+\kappa_-}{\kappa_- - \kappa_+}.
\tag{2.17}
$$

Following the definition of the local vector fields in [4–6]:

$$
\begin{aligned}
H_1 &= i\partial_+, \\
H_0 &= i\left(w^+\,\partial_+ + \frac{1}{2}y\,\partial_y\right), \\
H_{-1} &= i\left(w^{+2}\,\partial_+ + w^+ y\,\partial_y - y^2\,\partial_-\right),
\end{aligned}
\tag{2.18}
$$

and

$$
\begin{aligned}
\bar{H}_1 &= i\partial_-, \\
\bar{H}_0 &= i\left(w^-\,\partial_- + \frac{1}{2}y\,\partial_y\right), \\
\bar{H}_{-1} &= i\left(w^{-2}\,\partial_- + w^- y\,\partial_y - y^2\,\partial_+\right),
\end{aligned}
\tag{2.19}
$$

we can obtain the two sets of $SL(2,R)$ Lie algebra with respect to $(H_0, H_{\pm1})$ and $(\bar{H}, \bar{H}_{\pm1})$, respectively:

$$
[H_0, H_{\pm1}] = \mp i H_{\pm1}, \qquad [H_{-1}, H_1] = -2i H_0,
\tag{2.20}
$$

$$
\left[\bar{H}_0, \bar{H}_{\pm1}\right] = \mp i\bar{H}_{\pm1}, \qquad \left[\bar{H}_{-1}, \bar{H}_1\right] = -2i\bar{H}_0.
\tag{2.21}
$$

The corresponding quadratic Casimir operator of the $SL(2,R)$ Lie algebra reads as

$$
\begin{aligned}
\mathcal{H}^2 = \bar{\mathcal{H}}^2 &= -H_0^2 + \frac{1}{2}\left(H_1 H_{-1} + H_{-1} H_1\right) \\
&= \frac{1}{4}\left(y^2\,\partial_y^2 - y\,\partial_y\right) + y^2\,\partial_+\partial_-.
\end{aligned}
\tag{2.22}
$$

Thus, we rewrite the vector fields in terms of the coordinators (t, r, ϕ) as

$$H_1 = ie^{-2\pi T_R \phi} \left(\sqrt{\Delta}\partial_r + \frac{1}{2\pi T_R} \frac{r - \frac{1}{4\lambda_L}}{\sqrt{\Delta}}\partial_\phi + \frac{2T_L}{T_R} \frac{Mr - \frac{(a^2 + M^2)Q^2}{2M^2 - Q^2} - a^2}{\sqrt{\Delta}}\partial_t \right),$$

$$H_0 = \frac{i}{2\pi T_R}\partial_\phi + 2i\frac{1}{4\lambda_L}\frac{T_L}{T_R}\partial_t, \tag{2.23}$$

$$H_{-1} = ie^{2\pi T_R \phi} \left(-\sqrt{\Delta}\partial_r + \frac{1}{2\pi T_R} \frac{r - \frac{1}{4\lambda_L}}{\sqrt{\Delta}}\partial_\phi + \frac{2T_L}{T_R} \frac{Mr - \frac{(a^2 + M^2)Q^2}{2M^2 - Q^2} - a^2}{\sqrt{\Delta}}\partial_t \right),$$

and

$$\bar{H}_1 = ie^{-2\pi T_L \phi + 2\lambda_L t} \left(\sqrt{\Delta}\partial_r - \frac{a}{\sqrt{\Delta}}\partial_\phi - (2M - \frac{Q^2}{r})\frac{r}{\sqrt{\Delta}}\partial_t \right),$$

$$\bar{H}_0 = -2iM\partial_t, \tag{2.24}$$

$$\bar{H}_{-1} = ie^{2\pi T_L \phi - 2\lambda_L t} \left(-\sqrt{\Delta}\partial_r - \frac{a}{\sqrt{\Delta}}\partial_\phi - (2M - \frac{Q^2}{r})\frac{r}{\sqrt{\Delta}}\partial_t \right).$$

So, we obtain the corresponding Casimir operator

$$\mathcal{H}^2 = \partial_r \Delta \partial_r + \frac{2am(Q^2 - 2Mr)\omega + a^2(m^2 - 4M^2\omega^2)}{r(r - 2M) + a^2 + Q^2}$$
$$+ \frac{(Q^4 - 4M^2Q^2 + 8M^3r - 4MQ^2r)\omega^2}{r(r - 2M) + a^2 + Q^2}. \tag{2.25}$$

In the background of a KN black hole, the near region wave equation (2.15) can be rewritten as

$$\bar{\mathcal{H}}^2\Phi = \mathcal{H}^2\Phi = l(l+1)\Phi. \tag{2.26}$$

From the above procedure, we know that, in the near region, the scalar field can have $SL(2, R)_L \times SL(2, R)_R$ weight to obtain the microscopical entropy of the non-extremal KN black hole by the use of the Cardy Formula. However, we don't know how to compute the corresponding central charges c_L and c_R. Here, as did in [4], we also assume that the central charges of the non-extremal Kerr-Newman still keep same as the extremal case, i.e., $c_L = c_R = 12J$. Then, using the Cardy formula for the microstate, we have

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \pi(r_+^2 + a^2). \tag{2.27}$$

From the 2D dual CFT, we reproduce the KN black hole entropy in four dimensions which reaches agreement with the macroscopic entropy S_{BH}.

3. Scalar absorption

In this section we will calculate the absorption cross section for the KN black hole in the near region. The absorption cross section of a low-frequency massless scalar in the near-extremal limit of the KN black hole has been calculated and analyzed in [7–9]. In the near

region, the solutions of the radial wave equation (2.15) with respect to the ingoing and outgoing boundary conditions at the horizon are

$$
\begin{aligned}
R_{in}(r) = & \left(\frac{r-r_+}{r-r_-}\right)^{-i\frac{(\omega-m\Omega_H)}{4\pi T_H}} (r-r_-)^{-1-\ell} \\
& \times F\left(1+\ell-i\frac{4M^2-2Q^2}{r_+-r_-}\omega+i\frac{m\Omega_H}{2\pi T_H}, 1+\ell-i2M\omega; 1-i\frac{(\omega-m\Omega_H)}{2\pi T_H}; \frac{r-r_-}{r-r_+}\right),
\end{aligned}
\tag{3.1}
$$

and

$$
\begin{aligned}
R_{out}(r) = & \left(\frac{r-r_+}{r-r_-}\right)^{i\frac{(\omega-m\Omega_H)}{4\pi T_H}} (r-r_-)^{-1-\ell} \\
& \times F\left(1+\ell+i\frac{4M^2-2Q^2}{r_+-r_-}\omega-i\frac{m\Omega_H}{2\pi T_H}, 1+\ell+i2M\omega; 1+i\frac{(\omega-m\Omega_H)}{2\pi T_H}; \frac{r-r_-}{r-r_+}\right),
\end{aligned}
\tag{3.2}
$$

where $F(a,b;c;z)$ is the hypergeometric function. At the outer boundary of the matching region $r \gg M$ (but still $r \ll \frac{1}{\omega}$), the ingoing wave behaves as

$$
R_{in}(r \gg M) \sim Ar^\ell
\tag{3.3}
$$

with

$$
A = \frac{\Gamma(1-i\frac{\omega-m\Omega_H}{2\pi T_H})\Gamma(1+2\ell)}{\Gamma(1+\ell-i2M\omega)\,\Gamma(1+\ell-i\frac{4M^2-2Q^2}{r_+-r_-}\omega+\frac{i}{2\pi T_H}m\Omega_H)}\ .
\tag{3.4}
$$

The absorption cross section can be written as

$$
\begin{aligned}
P_{\text{abs}} \ \sim & \ |A|^{-2} \\
\sim & \ \sinh\left(\frac{\omega-m\Omega}{2T_H}\right)|\Gamma(1+\ell-i2M\omega)|^2 \\
& \times \left|\Gamma\left(1+\ell-i\frac{4M^2-2Q^2}{r_+-r_-}\omega+\frac{i}{2\pi T_H}m\Omega_H\right)\right|^2.
\end{aligned}
\tag{3.5}
$$

In order to match the absorption cross section of a near-region scalar field in the KN black hole background with the finite-temperature absorption cross section for the corresponding 2D CFT, we need study the first law of black hole thermodynamics. Since the existence of non-zero charge of the KN black hole, the first law of black hole thermodynamics is

$$
T_H\delta S = \delta M - \Omega_H \delta J - \varphi \delta Q,
\tag{3.6}
$$

where $\varphi = \frac{Q\,r_+}{r_+^2+a^2}$ is the electric potential of the KN black hole. It is obvious that δQ is equal to zero with respect to the uncharged scalar field. With the conjugate charges δE_R and δE_L, we can suppose the dual CFT entropy is

$$
\delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}.
\tag{3.7}
$$

The solution is

$$\delta E_L = \frac{2M^2 - Q^2}{a}\delta M \ ,$$

$$\delta E_R = \frac{2M^2 - Q^2}{a}\delta M - \delta J \ . \tag{3.8}$$

With the variations $\delta M = m$ and $\delta J = \omega$, the left and right moving frequencies are turned out to be

$$\omega_L \equiv \delta E_L = \frac{2M^2 - Q^2}{a}\omega \ ,$$

$$\omega_R \equiv \delta E_R = \frac{2M^2 - Q^2}{a}\omega - m. \tag{3.9}$$

By using of the above formula, we can rewrite the absorption cross section as

$$P_{\text{abs}} \sim T_L^{2h_L - 1} T_R^{2h_R - 1} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \left|\Gamma(h_L + i\frac{\omega_L}{2\pi T_L})\right|^2 \left|\Gamma(h_R + i\frac{\omega_R}{2\pi T_R})\right|^2 \ , \tag{3.10}$$

which agrees precisely with finite-temperature absorption cross section for a 2D CFT.

4. Conclusion

In this paper, the method of CMS is extended to the four-dimensional KN Black Hole in general relativity theory. We investigate the hidden conformal structure of the KN black hole by the low-frequency wave equation of a massless scalar in the near region. The dual CFT with the left and right temperatures $T_L = (2M^2 - Q^2)/(4\pi J)$ and $T_R = \sqrt{M^4 - J^2 - M^2 Q^2}/(2\pi J)$ is obtained. Furthermore, under the assumption that the central charges of the non-extremal Kerr-Newman still keep same as the extremal case, one can reproduce the macroscopic Bekenstein-Hawking entropy via the Cardy formula. At last, by rewriting the absorption cross section of a near-region scalar field as the finite-temperature absorption cross section for the 2D CFT, one can verify that the near-region KN black hole is dual to a 2D CFT.

From the result of this paper, we can see that the method of CMS is valid for the non-extremal Kerr-Newman black hole in 4D general relativity theory background. It would be interesting to investigate other charged and rotating black holes in diverse dimensions and in various gravity theories.

Acknowledgment

References

[1] M. Guica, T. Hartman, W. Song and A. Strominger, "The Kerr/CFT Correspondence," Phys. Rev. D **80**, 124008 (2009) [arXiv:0809.4266 [hep-th]].

[2] M. Cvetic and F. Larsen, "General rotating black holes in string theory: Greybody factors and event horizons," Phys. Rev. D **56**, 4994 (1997) [arXiv:hep-th/9705192]; M. Cvetic and D. Youm, "General Rotating Five Dimensional Black Holes of Toroidally Compactified Heterotic String," Nucl. Phys. B **476**, 118 (1996) [arXiv:hep-th/9603100]; S. W. Hawking, C. J. Hunter and M. Taylor, "Rotation and the AdS/CFT correspondence," Phys. Rev. D **59**, 064005 (1999) [arXiv:hep-th/9811056]; H. Lu, J. Mei, C. N. Pope and J. Vazquez-Poritz, "Extremal Static AdS Black Hole/CFT Correspondence in Gauged Supergravities," Phys. Lett. B **673**, 77 (2009) [arXiv:0901.1677 [hep-th]]; D. D. K. Chow, M. Cvetic, H. Lu and C. N. Pope, "Extremal Black Hole/CFT Correspondence in (Gauged) Supergravities," Phys. Rev. D **79**, 084018 (2009) [arXiv:0812.2918 [hep-th]]; A. M. Ghezelbash, "Kerr/CFT Correspondence in Low Energy Limit of Heterotic String Theory," JHEP **0908**, 045 (2009) [arXiv:0901.1670 [hep-th]]; T. Azeyanagi, N. Ogawa and S. Terashima, "The Kerr/CFT Correspondence and String Theory," Phys. Rev. D **79**, 106009 (2009) [arXiv:0812.4883 [hep-th]]; W. Y. Wen, "Holographic descriptions of (near-)extremal black holes in five dimensional minimal supergravity," arXiv:0903.4030 [hep-th]; C. M. Chen and J. E. Wang, "Holographic Duals of Black Holes in Five-dimensional Minimal Supergravity," arXiv:0901.0538 [hep-th].

[3] D. Astefanesei and H. Yavartanoo, "Stationary black holes and attractor mechanism," Nucl. Phys. B **794**, 13 (2008) [arXiv:0706.1847 [hep-th]]; R. A. Konoplya and A. Zhidenko, "Decay of a charged scalar and Dirac fields in the Kerr-Newman-de Sitter background," Phys. Rev. D **76**, 084018 (2007) [arXiv:0707.1890 [hep-th]]; B. Chen and Z. b. Xu, "Quasi-normal modes of warped black holes and warped AdS/CFT correspondence," JHEP **0911**, 091 (2009) [arXiv:0908.0057 [hep-th]]; B. Chen, B. Ning and Z. b. Xu, "Real-time correlators in warped AdS/CFT correspondence," JHEP **1002**, 031 (2010) [arXiv:0911.0167 [hep-th]]; B. Chen and C. S. Chu, "Real-time correlators in Kerr/CFT correspondence," arXiv:1001.3208 [hep-th]; T. Hartman, K. Murata, T. Nishioka and A. Strominger, "CFT Duals for Extreme Black Holes," JHEP **0904**, 019 (2009) [arXiv:0811.4393 [hep-th]]; D. Astefanesei and Y. K. Srivastava, "CFT Duals for Attractor Horizons," Nucl. Phys. B **822**, 283 (2009) [arXiv:0902.4033 [hep-th]]; A. M. Ghezelbash, "Kerr-Bolt Spacetimes and Kerr/CFT Correspondence," arXiv:0902.4662 [hep-th]; M. R. Garousi and A. Ghodsi, "The RN/CFT Correspondence," Phys. Lett. B **687**, 79 (2010) [arXiv:0902.4387 [hep-th]]; K. Hotta, "Holographic RG flow dual to attractor flow in extremal black holes," Phys. Rev. D **79**, 104018 (2009) [arXiv:0902.3529 [hep-th]]; T. Fukuyama, "SO(2,d-1) Gauge Theory of Gravity in d Dimensional Spacetime and AdS_d/CFT_{d-1} Correspondence," arXiv:0902.2820 [hep-th]; S. Nam and J. D. Park, "Hawking radiation from covariant anomalies in 2+1 dimensional black holes," Class. Quant. Grav. **26**, 145015 (2009) [arXiv:0902.0982 [hep-th]]; B. Chen and Z. b. Xu, "Quasinormal modes of warped AdS_3 black holes and AdS/CFT correspondence," arXiv:0901.3588 [hep-th]; F. Loran and H. Soltanpanahi, "5D Extremal Rotating Black Holes and CFT duals," Class. Quant. Grav. **26**, 155019 (2009) [arXiv:0901.1595 [hep-th]]; J. J. Peng and S. Q. Wu, "Extremal Kerr black hole/CFT correspondence in the five dimensional Gödel universe," Phys. Lett. B **673**, 216 (2009) [arXiv:0901.0311 [hep-th]]; H. Isono, T. S. Tai and W. Y. Wen, "Kerr/CFT correspondence and five-dimensional BMPV black holes," Int. J. Mod. Phys. A **24**, 5659 (2009) [arXiv:0812.4440 [hep-th]]; Y. Nakayama, "Emerging AdS from Extremally

Rotating NS5-branes," Phys. Lett. B **673**, 272 (2009) [arXiv:0812.2234 [hep-th]]; A. Garbarz, G. Giribet and Y. Vasquez, "Asymptotically AdS3 Solutions to Topologically Massive Gravity at Special Values of the Coupling Constants," Phys. Rev. D **79**, 044036 (2009) [arXiv:0811.4464 [hep-th]]; T. Azeyanagi, N. Ogawa and S. Terashima, "Holographic Duals of Kaluza-Klein Black Holes," JHEP **0904**, 061 (2009) [arXiv:0811.4177 [hep-th]]; G. W. Gibbons, C. A. R. Herdeiro, C. M. Warnick and M. C. Werner, "Stationary Metrics and Optical Zermelo-Randers-Finsler Geometry," Phys. Rev. D **79**, 044022 (2009) [arXiv:0811.2877 [gr-qc]]; K. Hotta, Y. Hyakutake, T. Kubota, T. Nishinaka and H. Tanida, "The CFT-interpolating Black Hole in Three Dimensions," JHEP **0901**, 010 (2009) [arXiv:0811.0910 [hep-th]]; M. Schvellinger, "Kerr-AdS black holes and non-relativistic conformal QM theories in diverse dimensions," JHEP **0812**, 004 (2008) [arXiv:0810.3011 [hep-th]]; M. Becker, S. Cremonini and W. Schulgin, "Extremal Three-point Correlators in Kerr/CFT," arXiv:1004.1174 [hep-th]; J. Mei, "The Entropy for General Extremal Black Holes," JHEP **1004**, 005 (2010) [arXiv:1002.1349 [hep-th]]; C. M. Chen, Y. M. Huang and S. J. Zou, "Holographic Duals of Near-extremal Reissner-Nordstrom Black Holes," JHEP **1003**, 123 (2010) [arXiv:1001.2833 [hep-th]]; J. J. Peng and S. Q. Wu, "Extremal Kerr/CFT correspondence of five-dimensional rotating (charged) black holes with squashed horizons," Nucl. Phys. B **828**, 273 (2010) [arXiv:0911.5070 [hep-th]]; C. Krishnan, "Tomograms of Spinning Black Holes," Phys. Rev. D **80**, 126014 (2009) [arXiv:0911.0597 [hep-th]]; C. M. Chen, J. R. Sun and S. J. Zou, "The RN/CFT Correspondence Revisited," JHEP **1001**, 057 (2010) [arXiv:0910.2076 [hep-th]]; J. Rasmussen, "A note on Kerr/CFT and free fields," arXiv:0909.2924 [hep-th]; D. Grumiller and A. M. Piso, "Exact relativistic viscous fluid solutions in near horizon extremal Kerr background," arXiv:0909.2041 [astro-ph.SR]; J. Rasmussen, "Isometry-preserving boundary conditions in the Kerr/CFT correspondence," Int. J. Mod. Phys. A **25**, 1597 (2010) [arXiv:0908.0184 [hep-th]]; Y. Matsuo, T. Tsukioka and C. M. Yoo, "Yet Another Realization of Kerr/CFT Correspondence," Europhys. Lett. **89**, 60001 (2010) [arXiv:0907.4272 [hep-th]]; Y. Matsuo, T. Tsukioka and C. M. Yoo, "Another Realization of Kerr/CFT Correspondence," Nucl. Phys. B **825**, 231 (2010) [arXiv:0907.0303 [hep-th]]; O. J. C. Dias, H. S. Reall and J. E. Santos, "Kerr-CFT and gravitational perturbations," JHEP **0908**, 101 (2009) [arXiv:0906.2380 [hep-th]]; L. M. Cao, Y. Matsuo, T. Tsukioka and C. M. Yoo, "Conformal Symmetry for Rotating D-branes," Phys. Lett. B **679**, 390 (2009) [arXiv:0906.2267 [hep-th]]; W. Kim and E. J. Son, "Central Charges in 2d Reduced Cosmological Massive Gravity," Phys. Lett. B **678**, 107 (2009) [arXiv:0904.4538 [hep-th]]; X. N. Wu and Y. Tian, "Extremal Isolated Horizon/CFT Correspondence," Phys. Rev. D **80**, 024014 (2009) [arXiv:0904.1554 [hep-th]]; C. Krishnan and S. Kuperstein, "A Comment on Kerr-CFT and Wald Entropy," Phys. Lett. B **677**, 326 (2009) [arXiv:0903.2169 [hep-th]]; T. Azeyanagi, G. Compere, N. Ogawa, Y. Tachikawa and S. Terashima, "Higher-Derivative Corrections to the Asymptotic Virasoro Symmetry of 4d Extremal Black Holes," Prog. Theor. Phys. **122**, 355 (2009) [arXiv:0903.4176 [hep-th]]; M. Cvetic and F. Larsen, "Greybody Factors and Charges in Kerr/CFT," JHEP **0909**, 088 (2009) [arXiv:0908.1136 [hep-th]]; T. Hartman, W. Song and A. Strominger, "Holographic Derivation of Kerr-Newman Scattering Amplitudes for General Charge and Spin," JHEP **1003**, 118 (2010) [arXiv:0908.3909 [hep-th]]; H. Lu, J. Mei and C. N. Pope, "Kerr/CFT Correspondence in Diverse Dimensions," JHEP **0904**, 054 (2009) [arXiv:0811.2225 [hep-th]]; I. Bredberg, T. Hartman, W. Song and A. Strominger, "Black Hole Superradiance From Kerr/CFT," JHEP **1004**, 019 (2010.) [arXiv:0907.3477 [hep-th]].

[4] A. Castro, A. Maloney and A. Strominger, "Hidden Conformal Symmetry of the Kerr Black Hole," arXiv:1004.0996 [hep-th].

[5] C. Krishnan, "Hidden Conformal Symmetries of Five-Dimensional Black Holes," arXiv:1004.3537 [hep-th].

[6] C.-M. Chen and J.-R. Sun, "Hidden Conformal Symmetry of the Reissner-Nordstrøm Black Holes," arXiv:1004.3963 [hep-th].

[7] J. M. Maldacena and A. Strominger, "Universal low-energy dynamics for rotating black holes," Phys. Rev. D **56**, 4975 (1997) [arXiv:hep-th/9702015].

[8] M. Cvetic and F. Larsen, "Greybody factors for rotating black holes in four dimensions," Nucl. Phys. B **506**, 107 (1997) [arXiv:hep-th/9706071].

[9] M. Cvetic and F. Larsen, "Greybody factors for black holes in four dimensions: Particles with spin," Phys. Rev. D **57**, 6297 (1998) [arXiv:hep-th/9712118].